BER analysis of collaborative dual-hop wireless transmissions

T.A. Tsiftsis, G.K. Karagiannidis, S.A. Kotsopoulos and F.-N. Pavlidou

The error performance of collaborative dual-hop wireless transmissions with maximal-ratio combining diversity is presented. Specifically, using the well-known inequality between geometric and harmonic mean of positive random variables, an upper bound for the end-to-end signal-to-noise-ratio is derived, and it is used to efficiently evaluate the average error probability.

Introduction: Recently, relaying dual-hop transmissions have gained a new lease of life in collaborative/co-operative wireless communication systems [1, 2]. In collaborative diversity systems, intermediate mobile terminals are used to relay the signal between the base station and the destination mobile terminal, when the direct link is in deep fade. Scanning the up-to-date open technical literature, the number of published works concerning performance analysis of dual-hop wireless communications systems with collaborative diversity is relatively small. In [1], an outage probability formula is derived using the method of multi-user spatial diversity. Later, Hasna and Alouini studied the outage and the error performance of dual-hop systems with regenerative and non-regenerative relays over Nakagami-m [2] and Rayleigh-fading channels [3]. In this Letter, using the well-known inequality between geometric and harmonic mean of positive random variables (RVs), we derive an upper bound for the end-to-end signal-to-noise ratio (SNR), which is used to evaluate in closed-form an efficient and tight lower bound for the error performance of collaborative dual-hop transmissions using maximal-ratio-combining (MRC) diversity in the destination mobile terminal.

System model: A multi-user wireless communications system, where the source terminal ***S*** communicates with the destination terminal ***D*** through a direct link with SNR γ_o and L dual-hop collaborative paths of non-regenerative (amplify and forward) relays, is considered in Fig. 1. Assuming MRC at the destination terminal, the overall SNR at the receiving end can be written as [2–4]:

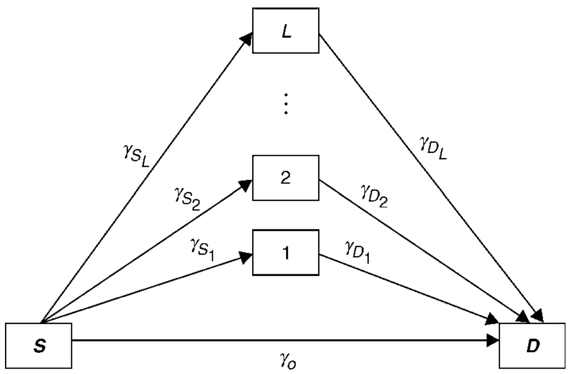

Fig. 1 *Wireless communication system where source **S** and destination **D** are communicating through L dual-hop collaborative diversity paths*

$$\gamma_{end} = \gamma_o + \sum_{i=1}^{L} \frac{\gamma_{S_i}\gamma_{D_i}}{\gamma_{S_i} + \gamma_{D_i} + 1} \tag{1}$$

where γ_{S_i} is the instantaneous SNR between the source ***S*** and relay i, and γ_{D_i} is the instantaneous SNR between the destination ***D*** and relay i.

Average error probability: The moment-generating function (MGF)-based approach [5, Chap. 1] for the performance evaluation of digital modulations over fading channels, allows us to obtain the average error probability for a wide variety of modulation schemes. Using (1), γ_{end} can be rewritten as:

$$\gamma_{end} = \gamma_o + \sum_{i=1}^{L} \frac{1}{1/\gamma_{S_i} + 1/\gamma_{D_i} + 1/\gamma_{S_i}\gamma_{D_i}} = \gamma_o + \sum_{i=1}^{L} \frac{H_i}{3} \tag{2}$$

where H_i is the harmonic mean of the three positive RVs γ_{S_i}, γ_{D_i} and $\gamma_{S_i}\gamma_{D_i}$, i.e. $H_i = 3(1/\gamma_{S_i} + 1/\gamma_{D_i} + 1/\gamma_{S_i}\gamma_{D_i})^{-1}$ for any path.

Using the well-known inequality between harmonic and geometric mean of positive RVs [6, p. 45]

$$H_i \leq G_i \tag{3}$$

with G_i being the geometric mean of γ_{S_i}, γ_{D_i} and $\gamma_{S_i}\gamma_{D_i}$, i.e. $G_i = (\gamma_{S_i}\gamma_{D_i}\gamma_{S_i}\gamma_{D_i})^{1/3}$, (2) results in:

$$\gamma_{end} \leq \gamma_b = \gamma_o + \frac{1}{3}\sum_{i=1}^{L}(\gamma_{S_i}\gamma_{D_i})^{2/3} \tag{4}$$

where γ_b is now an upper bound of γ_{end}, having the advantage of mathematical tractability over that in (1). Owing to the independency of γ_{S_i}, γ_{D_i} and $\gamma_{S_i}\gamma_{D_i}$, the MGF of γ_b equals the product of MGFs as

$$\mathcal{M}_{\gamma_b}(s) = \mathcal{M}_{\gamma_o}(s)\prod_{i=1}^{L}\mathcal{M}_{1/3(\gamma_{S_i}\gamma_{D_i})^{2/3}}(s) \tag{5}$$

where $\mathcal{M}_{\gamma_o}(s)$ and $\mathcal{M}_{(1/3)(\gamma_{S_i}\gamma_{D_i})^{2/3}}(s)$ are the MGFs of γ_o and $1/3(\gamma_{S_i}\gamma_{D_i})^{2/3}$, respectively.

Owing to the MGF definition, $\mathcal{M}_{\gamma_b}(s) \triangleq E\langle e^{s\gamma_b}\rangle$, (5) can be expressed as

$$\mathcal{M}_{\gamma_b}(s) = \mathcal{M}_{\gamma_o}(s) \times \prod_{i=1}^{L}\overbrace{\int_0^\infty\int_0^\infty e^{(s/3)\gamma_{S_i}^{2/3}\gamma_{D_i}^{2/3}} f_{\gamma_{S_i}}(\gamma_{S_i}) f_{\gamma_{D_i}}(\gamma_{D_i})\, d\gamma_{S_i}\, d\gamma_{D_i}}^{I} \tag{6}$$

Assuming a Nakagami-m fading environment, γ_{S_i} and γ_{D_i} are gamma distributed RVs with probability density function (pdf), $f_{\gamma_i}(\gamma_i)$, given by [5]:

$$f_{\gamma_i}(\gamma_i) = \frac{m_i^{m_i}}{\bar{\gamma}_i^{m_i}\Gamma(m_i)}\gamma_i^{m_i-1}e^{-m_i\gamma_i/\bar{\gamma}_i} \tag{7}$$

where $\Gamma(\cdot)$ is the gamma function [7, eqn. (8.310.1)], $\bar{\gamma}_i$ is the average SNR per hop and m_i is the Nakagami parameter describing the fading severity of the ith hop and assumed, with no loss of generality, to be the same in all hops.

Using (6) and (7), the first integral in I, i.e. the one on γ_{S_i}, is of the form

$$I_1 = \frac{1}{\Gamma(m)}\left(\frac{m}{\bar{\gamma}_{S_i}}\right)^m \int_0^\infty \gamma_{S_i}^{m-1} G_{0,1}^{1,0}\left(\frac{m}{\bar{\gamma}_{S_i}}\gamma_{S_i}\,\middle|\, \begin{matrix}-\\0\end{matrix}\right) \times G_{0,1}^{1,0}\left(-\frac{s\gamma_{D_i}^{2/3}}{3}\gamma_{S_i}^{2/3}\,\middle|\,\begin{matrix}-\\0\end{matrix}\right) d\gamma_{S_i} \tag{8}$$

where

$$G_{p,q}^{m,n}\left(x\,\middle|\,\begin{matrix}a_1,\ldots,a_p\\b_1,\ldots,b_q\end{matrix}\right)$$

is the Meijer's G-function [7, Chap. 9.3] and $e^{-m\gamma_{Si}/\bar{\gamma}_{Si}}$, $e^{(s/3)\gamma_{Si}^{2/3}\gamma_{Di}^{2/3}}$ are expressed in terms of the G-function [8]. Using [8], the integral I_1 can be evaluated in closed-form as:

$$I_1 = \frac{\sqrt{3}2^{m-1/2}}{(2\pi)^{3/2}\Gamma(m)} G_{2,3}^{3,2}\left[-\frac{4s^3}{3^6}\left(\frac{\bar{\gamma}_{S_i}}{m}\right)^2\gamma_{D_i}^2\,\middle|\,\begin{matrix}\frac{1-m}{2},\frac{2-m}{2}\\0,\frac{1}{3},\frac{2}{3}\end{matrix}\right] \tag{9}$$

The second integral in I, i.e. the one on γ_{D_i}, can be solved in the same way as I_1, resulting in:

$$I_2 = \frac{\sqrt{3}2^{2m-3}}{\pi^2\Gamma^2(m)} \times G_{4,3}^{3,4}\left(-\frac{2^4s^3}{3^6}\left(\frac{\bar{\gamma}_{S_i}\bar{\gamma}_{D_i}}{m^2}\right)^2\,\middle|\,\begin{matrix}\frac{1-m}{2},\frac{2-m}{2},\frac{1-m}{2},\frac{2-m}{2}\\0,\frac{1}{3},\frac{2}{3}\end{matrix}\right) \tag{10}$$

Using the expression for the MGF of γ_o [5], $\mathcal{M}_{\gamma_b}(s)$ can be finally written as:

$$\mathcal{M}_{\gamma_b}(s) = \left(1 - \frac{s\bar{\gamma}_o}{m}\right)^{-m} \prod_{i=1}^{L} \frac{\sqrt{3}2^{2m-3}}{\pi^2\Gamma^2(m)} \times G_{4,3}^{3,4}\left(-\frac{2^4 s^3}{3^6}\left(\frac{\bar{\gamma}_{S_i}\bar{\gamma}_{D_i}}{m^2}\right)^2 \middle| \begin{matrix} \frac{1-m}{2}, \frac{2-m}{2}, \frac{1-m}{2}, \frac{2-m}{2} \\ 0, \frac{1}{3}, \frac{2}{3} \end{matrix} \right) \tag{11}$$

For identical links, i.e. $\bar{\gamma}_o = \bar{\gamma}_{S_i} = \bar{\gamma}_{D_i} = \bar{\gamma}$ for $i = 1, 2, \ldots, L$, (11) can be written as:

$$\mathcal{M}_{\gamma_b}(s) = \left(1 - \frac{s\bar{\gamma}}{m}\right)^{-m} \left[\frac{\sqrt{3}2^{2m-3}}{\pi^2\Gamma^2(m)} G_{4,3}^{3,4} \times \left(-\frac{2^4 s^3}{3^6}\frac{\bar{\gamma}^4}{m^4} \middle| \begin{matrix} \frac{1-m}{2}, \frac{2-m}{2}, \frac{1-m}{2}, \frac{2-m}{2} \\ 0, \frac{1}{3}, \frac{2}{3} \end{matrix} \right) \right]^L \tag{12}$$

Having the MGF of γ_b in closed-form, as given in (12), and using the MGF-based approach for the performance evaluation of digital modulations over fading channels [5, Chap. 1], the average bit and symbol error rate can be evaluated for a wide variety of M-ary modulations (such as M-ary phase-shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM)).

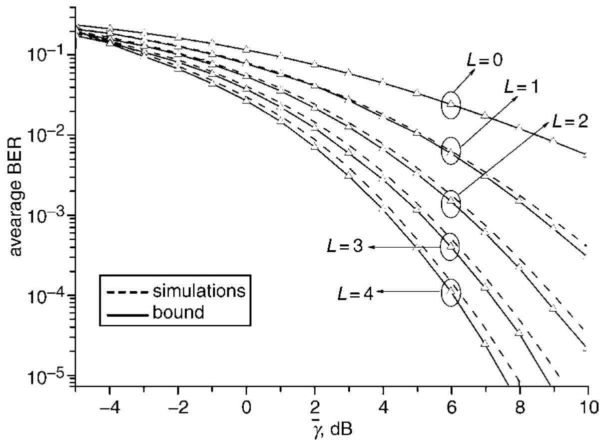

Fig. 2 *Error performance of BPSK for several numbers of collaborative diversity paths*

Numerical and simulation results: Fig. 2 shows the effect of the number of dual-hop collaborative diversity paths on the overall error performance, where BPSK modulation is considered. Curves for the exact error performance are also presented using Monte-Carlo simulations. It is evident that the performance is improved as the number of collaborative paths increases. In addition, the bound proposed in this Letter is more efficient, especially at low SNRs. It is emphasised here that, in the forthcoming generations of mobile wireless systems, almost 40% of users will experience receiver SNR levels below 0 dB while less than 10% will display levels above 10 dB [9].

Conclusions: An efficient lower bound to the average BER performance of collaborative dual-hop wireless transmissions with MRC diversity in the destination terminal is presented, by applying the well-known inequality of geometric and harmonic mean of RVs. Numerical and simulation results show the tightness of the proposed bound.

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References

1 Emamian, V., and Kaveh, M.: 'Combating shadowing effects for systems with transmitter diversity by using collaboration among mobile users'. Proc. Int. Symp. on Communications ISC'01, Taiwan, November 2001, pp. 1051–105.4

2 Hasna, M.O., and Alouini, M-S.: 'Application of the harmonic mean statistics to the end-to-end performance of transmission systems with relays'. Proc. IEEE Global Telecommunications Conf. GLOBECOM '02, Taipei, Taiwan, November 2002, pp. 1310–1314

3 Hasna, M.O., and Alouini, M-S.: 'End-to-end performance of transmission systems with relays over Rayleigh-fading channels', *IEEE Trans. Wirel. Commun.*, 2003, **2**, pp. 1126–1131

4 Laneman, J.N., and Wornell, G.W.: 'Energy efficient antenna sharing and relaying for wireless networks'. Proc. IEEE Wireless Communication and Networking Conf. WCNC'00, Chicago, October 2000, pp. 7–12

5 Simon, M.K., and Alouini, M-S.: 'Digital communication over fading channels' (Wiley, New York, 2000, 1st edn.)

6 Stuart, A., Ord, K., and Kendall, K.: 'Advanced theory of statistics, Vol. I: Distribution theory' (Edward Arnold, 1994, 6th edn.)

7 Gradshteyn, I.S., and Ryzhik, I.M.: 'Table of integrals, series, and products' (Academic, New York, 1994, 5th edn.)

8 Adamchik, V.S., and Marichev, O.I.: 'The algorithm for calculating integrals of hypergeometric type functions and its realization to REDUCE system'. Proc. Int. Conf. on Symbolic and Algebraic Computation, Tokyo, Japan, 1990, pp. 212–224

9 Lozano, A., Tulino, A.M., and Verdu, S.: 'Multiple-antenna capacity in the low-power regime', *IEEE Trans. Inf. Theory*, 2003, **49**, pp. 2527–2544