PERFORMANCE OF DS/CDMA SYSTEMS WITH DIFFERENTIAL *M*-ARY ORTHOGONAL MODULATION AND RS-CODING FOR LEO SATELLITE COMMUNICATIONS

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SUMMARY

This paper presents a coded modulation scheme based on *M*-ary orthogonal modulation by means of Walsh–Hadamard (WH) sequences, suitable for low-earth-orbit (LEO) direct sequence/code division multiple access (DS/CDMA) satellite communication systems. Based on the IS-95 scheme, we consider Reed–Solomon (RS)-coded *M*-ary orthogonal modulation with error or erasures decoding, which presents good performance enhancement with low complexity. LEO satellite links are characterized by large Doppler frequency shifts caused by the difference in velocity between the satellite and the earth mobile terminal, which make conventional non-coherent detection ineffective. In order to overcome the phase shift variations during the symbol period, which result in orthogonality loss of the WH sequences, we applied a differential encoding process to the spreading sequences or the WH chips prior to transmission. A special diversity process suitable for the environment under consideration is also applied. Simulation results show that the proposed diversity/coding/modulation scheme attains very good performance at low transmitter/receiver complexity. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: differential encoding; DS/CDMA; LEO satellite communications; RS codes; *M*-ary orthogonal modulation

1. INTRODUCTION

M-ary orthogonal modulation by means of WH sequences is a well-established scheme for DS/CDMA applications. It has been investigated in several configurations for terrestrial cellular mobile, either indoor or outdoor, and LEO satellite communications.1-10 The increased spectral efficiency that *M*-ary orthogonal modulation offers, compared with conventional DS/CDMA systems, has placed it among the most promising schemes for future wireless communications. In conjunction with proper modulation, effective channel coding is very important for CDMA systems in order to achieve higher bandwidth efficiency and user capacity. Lately, RS-coded M-ary orthogonal modulation with error decoding has been proposed and found to present good performance with low complexity (bounded distance decoding).^{1,2} In this paper, erasure decoding is also considered resulting in further performance enhancement without significant complexity increase, owing to a simple erasure criterion.

The special character of the LMS (land mobile satellite) channel originates crucial problems in the

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application of *M*-ary orthogonal modulation. The most important one is the large performance degradation that M-ary orthogonal modulation faces in high-Doppler-shift environments such as the LMS channel when envelope detection is applied. This is due to the phase shift of the transmitted waveforms, which destroys orthogonality at the receiver.10,11 Thus envelope detection without any special device to remove the Doppler shift (which takes values of the order of the symbol rate or even higher) is applicable only for low Doppler shifts (lower than 0.3 times the symbol rate). Instead of using such a device and in order to keep the complexity and cost as low as possible, we have recently applied two techniques based on chip-by-chip differential encoding. Chip-by-chip differential encoding after spreading was first proposed in Reference 12. In this paper the differential process is applied either to the resulting wave-form after coding and spreading (DDS *M*-ary)¹³ or at the WH chip level (D*M*ary).¹⁴ This allows effective non-coherent detection when the Doppler shift is lower than the chip rate. Thus the system performs almost independently of the Doppler shift, but this is achieved at the expense of the signal-to-noise-plus-interference ratio (SNIR) increase that is required to attain the desired bit error probability (BEP).

In high-Doppler-shift environments where the path amplitude may vary during the symbol period, the selection or combination of two or more diversity

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branches must be properly designed. In order to overcome the high amplitude variability, we propose a diversity scheme based on the maximum output selection scheme, where the diversity branches are scanned at a higher rate than the symbol rate. In this way the averaging process of the amplitude that is implicit in symbol-by-symbol discrimination is avoided.

The performance prediction of the proposed systems is done via simulation. This was necessary because theoretical evaluation is rather sophisticated when the special characteristics of the LMS channel are taken into account. Channel simulation was based on a deterministic model, recently proposed in Reference 15, properly adapted to include the LMS channel characteristics.

2. SYSTEM MODEL

It is assumed that the system under consideration operates in a single spot beam of a satellite. Interference from other spot beams or users communicating with other satellites is ignored for simplicity. Power control is applied and is considered to be perfect in the sense of completely removing the large-scale slow fading.^{16–18}

2.1. Transmitter

The transmitter of the proposed RS-coded DS/CDMA system is shown in Figure 1. Figure 1(a) illustrates the conventional scheme without differential encoding, Figure 1(b) the DDS M-ary scheme with differential encoding applied at the spreading sequence (SS) chips and Figure 1(c) the DM-ary scheme with differential encoding applied at the WH chips. In all cases, b source information bits of rate r_b are grouped and mapped to one of the $M = 2^{b}$ information symbols. The information symbols of rate $r_s = r_b/\log_2 M$ symbols per second enter an (n,k) RS encoder operating in GF(M), with n and k the block length and the number of information symbols each block contains respectively. The output coded symbols of rate $r_{sc} = r_s/r_c$, where $r_{\rm c} = k/n$, are interleaved and then mapped to one of the M WH orthogonal sequences.

When no differential encoding is applied, the WH chips are oversampled by a factor L = N/M (integer) and then multiplied with an *N*-length random spreading sequence. The resulting samples modulate a T_c -energy chip wave-form g(t). The equivalent lowpass transmitted signal of the *k*th user is given by

$$u_m^{(k)}(t) = \sum_{n=0}^{N-1} c_n^{(k)} g(t - nT_c)$$
(1)

with

$$c_n^{(k)} = h_{\lfloor n/L \rfloor, m}^{(k)} a_n^{(k)}$$
(2)



(c)

Figure 1. Transmitter configurations of (a) conventional, (b) DDS *M*-ary and (c) D*M*-ary schemes

where $h_{i,m}^{(k)}$ is the *i*th WH chip amplitude of the *m*th WH sequence of the *k*th user and $a_n^{(k)}$ is the *n*th chip amplitude of the spreading sequence of the *k*th user.

For the D*M*-ary scheme¹⁴ the WH chips are differentially encoded and then oversampled and spread, i.e.

$$c_n^{(k)} = \beta_{n/L}^{(k)} a_n^{(k)} \tag{3}$$

where $\beta_{ln/L}^{(k)}$ are the oversampled differentially encoded WH chips of the *k*th user with period T_H generated by

$$\beta_{i}^{(k)} = \beta_{i-1}^{(k)} h_{i,m}^{(k)} \tag{4}$$

When differential encoding is applied at the

spreading sequence chip level (DDS M-ary),¹³ the samples

$$\beta_n^{(k)} = h_{\lfloor n/L \rfloor, m}^{(k)} a_n^{(k)} \tag{5}$$

after oversampling and spreading are differentially encoded at the SS chip rate and generate the binary symbols

$$c_n^{(k)} = c_{n-1}^{(k)} \beta_n^{(k)} \tag{6}$$

In all three cases the total bandwidth expansion factor introduced by the modulator and the encoder is

$$\eta = \frac{W}{r_b} = \frac{N}{r_c \log_2 M} \tag{7}$$

where *W* is the single-sided occupied spectrum. Assuming a constant bandwidth expansion factor, several configurations may be derived by choosing a symbol set size *M* and then adjusting r_c and *N* so that *N/M* is an integer. Extended or shortened RS codes may arise,^{1,2} some of which are given in Table 1 for $\eta = 128$.

2.2. Channel Model

The channel model used is identical to the one used in References 16 and 18, coming out with a modification of the model described in Reference 19. The received amplitude may follow a Rice or a Rayleigh distribution depending on the existence or not of a line-of-sight (LOS) path. The lognormal process describing the large-scale variations of the local mean power in the non-line-of-sight (NLOS) case was not included because of the perfect slow power control procedure assumed that eliminates it. Thus the mixed envelope probability distribution function is given by

$$f(x) = B \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$+ (1 - B) \frac{x}{\sigma^2} \exp\left(-\frac{x^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{xA}{\sigma^2}\right)$$
(8)

where $2\sigma^2$ is the mean power of the diffuse component and *A* is the amplitude of the specular component. The Rice factor is $R = A^2/2\sigma^2$ and was taken equal to 7 dB for the simulation. *B* is the time-share shadowing parameter determining the state of the channel, namely 'good' for the LOS and 'bad' for the NLOS case, given by

$$B = \frac{D_b}{D_g + D_b} \tag{9}$$

where D_g and D_b are the mean durations of the good and the bad state of the channel respectively. The specific values of D_g and D_b used in the simulations were 9 m and 70 m respectively and they were selected from measurement results¹⁹ corresponding to a city environment with an elevation angle of 13°. In the simulations the two states were driven by a two-state Markov chain. The Rayleigh process was developed using a deterministic model (described in Reference 15) that generates a complex random process whose components (real and imaginary) are zero-mean independent Gaussian processes with variance σ^2 and autocorrelation function $r(t) = \sigma^2 J_0(2\pi f_d t)$, where $J_0(\cdot)$ is the zeroth-order Bessel function and f_d is the maximum Doppler frequency. For compactness and generalization the normalized Doppler frequency shift

$$\Delta f_d = f_d / r_{sc}$$

is used for the presented results. The Rice process was developed similarly by adding the LOS amplitude to the real part of the complex random process mentioned above. In either case (Rice or Rayleigh) the fading is assumed to be non-selective. Figure 2 presents a block diagram of the channel model.

2.3. Receiver Model

The receivers' structure is shown in Figure 3. They consist of two major parts: the detector and the decoder. The conventional detector without differential encoding (Figure 3(a)) consists of a filter matched to the chip wave-form, followed by a sampling device. Perfect clock recovery is assumed so



Figure 2. Channel model

Table I. Some possible system configurations with $\eta = 128$

М	16	16	32	32	64	64
(<i>n</i> , <i>k</i>)	(16,4)	(16,8)	(30,12)	(30,24)	(63,21)	(63,42)
Ν	128	256	256	512	256	512



Figure 3. Detector configurations of (a) conventional, (b) DDS M-ary and (c) DM-ary schemes

that interchip interference is eliminated. The complex-valued samples r_n are multiplied with the locally generated spreading sequence and then pass through a bank of envelope correlators corresponding to the M orthogonal WH sequences. The Moutputs $z_m, m = 1, ..., M$, are the final decision variables. Figure 3(b) illustrates the DDS M-ary detector. The complex-valued samples r_n are differentially decoded and then despreading and correlation are applied. For the DM-ary scheme (Figure 3(c)), differential decoding takes place after summation over the L SS chips that produce the estimated differentially encoded WH chips β'_i . The differentially decoded WH chips, i.e. $h_{i,m}' = \beta'_i \beta^{*'}_{i-1}$, are used as input to the correlators.

In order to deal with high Doppler frequency shift and remove its effects, the differential encoding procedure should be performed in a time basis that assures stability of the channel phase. This fact led¹² to the use of SS chip-by-chip differential encoding. As the spreading gain gets higher, the variability of

the channel during the SS chip duration is reduced. The drawback is that splitting the differential encoding/decoding process among more chips during the symbol interval results in higher non-coherent combining loss. This was evident in Reference 12. In order to have successful differential decoding and remove the channel phase, we need a differential encoding rate much higher than the Doppler frequency shift. The WH chip rate is almost always lower than the SS chip rate and for the channel under consideration is higher than the Doppler frequency shift. Thus, by moving the differential encoding process from the SS to the WH chips, the differential combining loss is lowered, while effective channel phase removal is secured when large *M*-ary orthogonal sets are used (e.g. M = 64). Therefore the DM-ary scheme is expected to perform better than the DDS *M*-ary scheme for the Doppler frequency shifts under consideration.

After deinterleaving, the *M* decision variables are used for the decoding process. When only errors decoding is applied, the maximum decision variable is considered to correspond to the correct symbol, so mapping to this symbol over GF(*M*) is applied. The decoder is capable of correcting $\lfloor \frac{(n-k)}{2} \rfloor$ errors.

When both error and erasure decoding are applied, an erasure criterion should be considered. A commonly used criterion decides that the detected symbol is an erasure if the mean, during a certain period, channel amplitude is below a certain value.²⁰ This technique can be applied only if an amplitude estimator is used at the receiver. We used a simpler, yet satisfying, rule for the erasure determination without channel state information, as proposed in References 13 and 14. The two greater decision variables obtained by the detector, i.e. z_m and z_p , with $z_m > z_p$, are subtracted and the result is normalized with respect to the maximum, namely z_m . When the result is lower than a certain value, the symbol is considered to be an erasure. Thus, if $(z_m - z_p)/z_m < \rho$, where ρ is a preset value, an erasure is flagged. The decoder is capable of correcting s errors and ε erasures provided that $2s + \varepsilon \leq n - k$.

2.4. Diversity Considerations

Diversity is a standard technique for enhancing the performance of the system. The conventional system, i.e. without differential encoding, has presented high performance gain with the use of maximum output selection diversity (MOSD)²¹ for terrestrial communications.^{1,2} Moreover, MOSD is a very attractive scheme because no channel amplitude or SNIR information is necessary, thus keeping the receiver complexity low.

The application of diversity to the system under consideration must be done carefully to secure performance enhancement. For low Doppler frequency shift the channel amplitude is almost constant throughout the symbol duration, i.e. the Hadamard sequence. Thus the selection of the diversity branch to account for demodulation may take place at the symbol level, i.e. between the final decision variables z_m of each diversity branch. On the other hand, high Doppler frequency shift leads to significant variability of the channel amplitude in the symbol duration (Figure 4). Selecting the maximum variable z_m between diversity branches is unreliable since the channel peaks and troughs are averaged during the calculation of z_m . Therefore diversity branch scanning should take place at time intervals shorter than the symbol duration to yield performance gain. Regarding LEO satellite communications, where Doppler shifts usually take values beyond the transmitted symbol rate, diversity at the chip level will be more effective than conventional diversity at the symbol level.

3. RESULTS AND DISCUSSION

Among the various configurations presented in Table 1. the M = 64, N = 512, RS (63.42) scheme was selected for comparing conventional, DM-ary and DDS M-ary signalling schemes. Two reasons led to this selection. The selected coded/modulation configuration was found to present the best performance for terrestrial applications^{1,2} including both Rayleigh and Rice fading environments. Insofar as the LEO satellite model under consideration refers to the same fading processes, it is expected that the performance of the selected configuration will remain the best among the other ones. Compatibility with the IS-95 configuration for voice communications, which uses an M = 64 WH signal set, was the second reason. Towards this fact, the SS chip rate under consideration is 1.2288 kchips/s, while the WH chip rate is 153.6 kchips/s. This choice validates the non-selective nature of the channel, which presents a coherence bandwidth of several MHz.

In order to simplify the simulation process and focus on the effect of the channel characteristics, the multi-user interference inherent in DS/CDMA systems was assumed to act as an additional source of additive Gaussian noise. This assumption is valid for long spreading sequences, such as the ones under consideration, and a significant number of simultaneous users. Therefore the BEP results to be presented are given as a function of the SNIR.

Figures 5–8 present simulation results, in terms of BEP versus SNIR for the two differentially encoded proposed systems and the conventional system,



Figure 4. Normalized received signal amplitude with $\Delta f_d = 1.0$

under a wide range of Doppler frequency shifts. The Doppler shifts simulated were selected in a wide range of the transmission symbol rate, i.e. $\Delta f_d \equiv f_d/r_{sc} = 0.025, 0.25, 0.5$ and 1. Higher Doppler shifts may also appear in the LEO satellite channel. However, the results to be presented show that in such a case and provided that the Doppler shift remains well below the chip rate, the performance will not change significantly. On the other hand, this Doppler shift range was selected in order to make clear the necessity of a modulation/coding scheme resistant to high Doppler frequency shifts. In all cases a block interleaver operating at the symbol level with length 63 (equal to the coded word length) and depth 40 was used.

Figure 5 presents BEP results of the three schemes under consideration, i.e. the conventional and chipby-chip differentially encoded systems, over the LMS channel with normalized Doppler shift $\Delta f_d = 0.025$. The conventional system outperforms the differentially encoded systems in all three cases of uncoded and coded with error or erasure decoding. The D*M*-ary configuration clearly performs better than the DDS *M*-ary one. A difference of the order of 3–4 dB is evident between the two differentially encoded schemes. Erasure decoding offers an extra gain of about 1 and 2 dB for the conventional and differentially encoded systems respectively at a BEP of 10^{-3} .

Figure 6 shows simulation results for a normalized Doppler shift of 0.25. The conventional system's performance is degraded, but it still remains better than that of the differentially encoded schemes for low SNIR values. On the other hand, the DM-ary and DDS *M*-ary systems' performance is getting better. This is due in part to the fact that the channel



Figure 5. BEP results of conventional, DDS *M*-ary and D*M*-ary schemes for $\Delta f_d = 0.025$



Figure 6. BEP results of conventional, DDS *M*-ary and D*M*-ary schemes for $\Delta f_d = 0.25$

amplitude is averaged, as previously noted, for higher Doppler shifts, thus reducing the effect of deep fades. Additionally, higher Doppler shifts come with faster channel amplitude fluctuations and thus randomize detection errors. This results in a better decoder performance under a fixed interleaving depth.

Figures 7 and 8 present the simulation results for $\Delta f_d = 0.5$ and 1 respectively. The necessity of reducing the Doppler effect, either with a special device or with differential encoding, is clear. The conventional system's performance is degraded rapidly with Doppler increase and becomes unacceptable for $\Delta f_d = 1$. This is true for both coded and uncoded cases. Even with erasure decoding, the conventional



Figure 8. BEP results of conventional, DDS *M*-ary and D*M*-ary schemes for $\Delta f_d = 1$

system's performance is far worse than that of the error decoding case of the differentially encoded systems for BEPs of interest at $\Delta f_d = 0.5$. On the other hand, the 3 dB difference between the D*M*-ary and DDS *M*-ary systems remains almost unchanged. This is expected to be continued for higher Doppler shifts as well. The performance of the differentially encoded schemes will improve until the Doppler shift becomes so large that the phase varies significantly during the chip interval. This will be evident for the D*M*-ary scheme first of course.

Figure 9 presents simulation results of the D*M*ary scheme with error and erasure decoding for several normalized Doppler frequency shift values, with MOSD of second order or without diversity.



Figure 7. BEP results of conventional, DDS *M*-ary and D*M*-ary schemes for $\Delta f_d = 0.5$



Figure 9. BEP results of DM-ary scheme with and without diversity

Obviously, diversity enhances system performance in all Doppler frequency shift cases. It should be mentioned that diversity is more advantageous for lower Doppler spread values. This is due to the fact that with WH chip-based diversity the averaging process implicit in the calculation of the final decision variables is significantly removed, as more than one channel branch is selected during the symbol duration. Therefore the system performs almost identically for all Doppler frequency shifts. It must be mentioned though that conventional MOSD with a symbol time basis decision will yield better results for low Doppler shifts. Square law combining at the WH chip rate was also considered (omitted here for the sake of space) and found to present slightly worse performance than MOSD.

4. CONCLUSIONS

This work proposes a differential encoding technique for DS/CDMA with M-ary orthogonal signalling suitable for high-Doppler-spread environments such as the LEO mobile satellite communication channel. The differential encoding procedure is applied at the WH chip level (DM-ary scheme) or at the spreading sequence chip level (DDS M-ary scheme). At high Doppler shifts the performance of the differentially encoded schemes improves, while the conventional system's performance is degraded rapidly. A 3-4 dB gain of the DM-ary scheme over the DDS M-ary scheme was evidenced in all cases. A diversity technique based on the maximum output diversity selection scheme and suitable for high-Doppler-shift environments has also been proposed. The selection between diversity branches is performed in a WH chip time basis rather than in a symbol time basis, leading to performance independent of the Doppler shift. Moreover, RS encoding with erasure decoding offers a 2 dB gain at least in all examined cases without increasing receiver complexity, owing to a simple erasure criterion presented. The channel model used for the simulations corresponds to a hostile city environment and further results should be obtained for various channel models in order to establish the effectiveness of the proposed scheme.

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