Two-Dimensional Traffic Models for Cellular Mobile Systems

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Abstract— Two-dimensional traffic models for cellular mobile systems are formulated and the main performance measures are evaluated. System analysis in its general form is rather complex but a solution is always attained in closed form or by numerical techniques.

I. INTRODUCTION

TRAFFIC in a telecommunications network is traditionally classified as voice and data but the present technology advances promise and video services in the near future over broadband channels. Moreover land wired communications networks will integrate new mobile services. Recently great attention has been paid to these mobile services, especially in cellular systems which will soon cover urban areas. A lot of topics concerning these systems have been studied, i.e. frequency assignment techniques, channel access methods, transmission quality, standards for interfacing with the wired network, traffic analysis e.t.c. Especially, referring to the last topic, some mathematical models have been developed adequate for the evaluation of the main performance measures of voice systems [1], [2]. Traffic due to vehicular as well as portable terminals has been studied. But as far as we know there are no models covering the case of mixed media -voice, data, video-cellular systems. Since traffic analysis in mobile mixed service networks is a really hard problem, we shall leave aside the broadband services and we'll try to analyze voice-data mobile networks.

In the past some operation research scientists tried to solve mixed traffic M/M/S systems [3], [4]. Mathematical models have been formulated for nonpreemtive as well as preemptive priority of one kind of traffic over the other. The main parameters of system performance as the blocking and delay probabilities and the mean time delay in the queue were evaluated.

In this work we try to extend these models in purpose of covering voice-data cellular systems. The mathematical model and parameter evaluation are presented in the following sections.

II. MATHEMATICAL MODEL AND ANALYSIS

In an one-dimensional mobile system with N cells in series (highway) each one of length L the base stations are usually attached in the center of the cells. Four Poisson arrival streams are entering each cell, that is originating new voice calls, originating new data packets, handoff voice calls and handoff data packets, with rates $\lambda_{nv}, \lambda_{np}, \lambda_{hv}, \lambda_{hp}$

respectively. A set of C channels are available for these arrivals in a complete sharing scheme in FIFO order. If there is no available free channel new or handoff calls as well as new packet arrivals are blocked while handoff packets are placed in an infinite queue in FIFO order. Up to this point the system description is not different from classical M/M/S systems but there is one factor characterising cellular systems. The mobility noticed in the queue stream. A handoff packet being in queue may cross a cell without getting service because the vehicle must leave the cell. Thus the handoff packet may be transferred from one queue to another. The time a data packet will stay in the queue depends not only on the system state but on the system structure, too, which means on vehicular speed and cell dimensions. We assume that the time T_Q a vehicle needs to cross a cell follows an exponential distribution with mean $1/\mu_Q$.

The channel holding time of a new or handoff voice call is assumed also to follow an exponential distribution with mean $1/\mu_v$ and the same holds for any kind of packet arrivals with mean $1/\mu_p$. These channel holding times in a cell are easily extracted by statistical measurements and they are different from the total service time of a communication because of handoffs. Moreover, studying the system performance it is clear that the channel holding times in a cell are more important than the unencumbered session durations.

Now we shall evaluate the performance state parameters and first of all the state probabilities $P_{i,j}$, that is the probability of being i voice calls and j data packets in the system. The state diagram model is given in Fig.1 for all possible region of states.

For $i = 0,1,2,...,C$ and $j = 0,1,2,...$ the system balanced equations are

$$\left[\lambda_{nv} + \lambda_{hv} + \lambda_{np} + \lambda_{hp} + j\mu_p + i\mu_v\right]P_{i,j} = (\lambda_{nv} + \lambda_{hv})P_{i-1,j} +$$

$$\left(\lambda_{np} + \lambda_{hp}\right)P_{i,j-1} + (i+1)\mu_v P_{i+1,j} +$$

$$(j+1)\mu_p P_{i,j+1} \tag{1}$$

for $i = 0,1,2,...,C-1$ and $j = 0,1,2,...,C-1-i$

$$\left[\lambda_{hp} + i\mu_v + (C-i)\mu_p\right]P_{i,C-i} = (\lambda_{nv} + \lambda_{hv})P_{i-1,C-i} +$$

$$\left(\lambda_{np} + \lambda_{hp}\right)P_{i,C-i-1} + (i+1)\mu_v P_{i+1,C-i} +$$

$$\left[\mu_Q + (C-i)\mu_p\right]P_{i,C-i+1} \tag{2}$$

for $i = 0,1,2,...,C$ and $j = C-i$

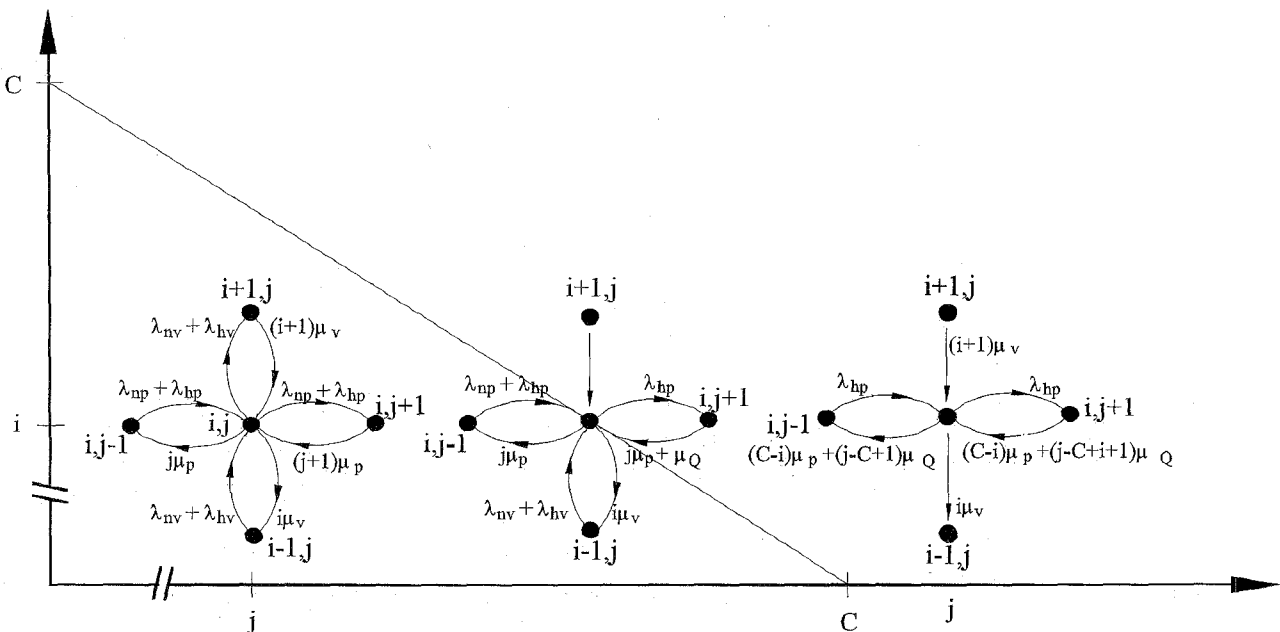$$\left[\lambda_{hp} + i\mu_v + (C-i)\mu_p + (j-C+i)\mu_Q\right]P_{i,j} =$$

Fig.1. Two dimensional diagram.

$$\lambda_{hp} P_{i,j-1} + (i+1)\mu_v P_{i+1,j} +$$

$$\left[(C-i)\mu_p + (j-C+i+1)\mu_Q \right] P_{i,j+1} \tag{3}$$

$$\text{for } i = 0,1,2,...,C \text{ and } j \geq C-i+1$$

We define the generating functions

$$\Pi_i(z) = \sum_{j=0}^{\infty} P_{i,j} z^j \qquad |z| \leq 1 \qquad i = 0,1,2,...,C$$

and then from (1)-(3) we have

$$\left[\lambda_{nv} + \lambda_{hv} + \lambda_{np} + \lambda_{hp} + j\mu_p + i\mu_v \right] P_{i,j} z^j =$$

$$\left(\lambda_{nv} + \lambda_{hv} \right) P_{i-1,j} z^j + \left(\lambda_{np} + \lambda_{hp} \right) P_{i,j-1} z^j +$$

$$(i+1)\mu_v P_{i+1,j} z^j + (j+1)\mu_p P_{i,j+1} z^j \tag{4}$$

$$\text{for } j = 0,1,2,...,C-i-1$$

$$\left[\lambda_{hp} + i\mu_v + (C-i)\mu_p \right] P_{i,C-i} z^{C-i} =$$

$$\left(\lambda_{nv} + \lambda_{hv} \right) P_{i-1,C-i} z^{C-i} + \left(\lambda_{hp} + \lambda_{np} \right) P_{i,C-i-1} z^{C-i} +$$

$$(i+1)\mu_v P_{i+1,C-i} z^{C-i} + \left[\mu_Q + (C-i)\mu_p \right] P_{i,C-i+1} z^{C-i} \tag{5}$$

$$\text{for } j = C-i$$

$$\left[\lambda_{hp} + i\mu_v + (C-i)\mu_p + (j-C+i)\mu_Q \right] P_{i,j} z^j =$$

$$\lambda_{hp} P_{i,j-1} z^j + (i+1)\mu_v P_{i+1,j} z^j +$$

$$\left[(C-i)\mu_p + (j-C+i+1)\mu_Q \right] P_{i,j+1} z^j \tag{6}$$

$$\text{for } j \geq C-i+1$$

Adding and subtracting the necessary terms we have

$$\left[\lambda_{hp} + i\mu_v + (C-i)\mu_p \right] \Pi_i(z) z +$$

$$z\left(\lambda_{nv} + \lambda_{hv} + \lambda_{np} \right) \sum_{j=0}^{C-i-1} P_{i,j} z^j +$$

$$z\mu_p \sum_{j=0}^{C-i-1} j P_{i,j} z^j - z(C-i)\mu_p \sum_{j=0}^{C-i-1} P_{i,j} z^j +$$

$$z\mu_Q \sum_{j=C-i+1}^{\infty} (j-C+i) P_{i,j} z^j =$$

$$z\left(\lambda_{nv} + \lambda_{hv} \right) \sum_{j=0}^{C-i} P_{i-1,j} z^j + \lambda_{hp} \Pi_i(z) z^2 +$$

$$z^2 \lambda_{np} \sum_{j=0}^{C-i-1} P_{i,j} z^j + z(i+1)\mu_v \Pi_{i+1}(z) +$$

$$(C-i)\mu_p \Pi_i(z) - (C-i)\mu_p \sum_{j=0}^{C-i-1} P_{i,j} z^j +$$

$$\mu_p \sum_{j=0}^{C-i-1} P_{i,j} z^j + \mu_Q \sum_{j=C-i+1}^{\infty} (j-C+i) P_{i,j} z^j \qquad (7)$$

After some manipulations we have

$$\left\{ \left[\lambda_{hp} + i\mu_v + (C-i)\mu_p \right] z - \lambda_{hp} z^2 - (C-i)\mu_p \right\} \Pi_i(z) =$$

$$(i+1)\mu_v z \Pi_{i+1}(z) +$$

$$\left[z^2 \lambda_{np} + (C-i)\mu_p z - z(\lambda_{nv} + \lambda_{hv} + \lambda_{np}) - (C-i)\mu_p \right] \sum_{j=0}^{C-i-1} P_{i,j} z^j +$$

$$\mu_p(1-z) \sum_{j=0}^{C-i-1} j P_{i,j} z^j + \mu_Q(1-z) \sum_{j=C-i+1}^{\infty} (j-C+i) P_{i,j} z^j +$$

$$z(\lambda_{nv} + \lambda_{hv}) \sum_{j=0}^{C-i} P_{i-1,j} z^j \qquad (8)$$

But

$$\mu_Q(1-z) \sum_{j=C-i+1}^{\infty} (j-C+i) P_{i,j} z^j = \mu_Q(1-z) \sum_{j=C-i+1}^{\infty} j P_{i,j} z^j -$$

$$\mu_Q(1-z)(C-i) \Pi_i(z) + \mu_Q(1-z)(C-i) \sum_{j=0}^{C-i+1} P_{i,j} z^j \qquad (9)$$

and

$$\mu_Q(1-z) \sum_{j=C-i+1}^{\infty} j P_{i,j} z^j = \mu_Q(1-z) \Pi_i'(z) z -$$

$$\mu_Q(1-z) \sum_{j=0}^{C-i+1} j P_{i,j} z^j \qquad (10)$$

Thus we result in a general recursive equation

$$\left[\left(\lambda_{hp} + i\mu_v + (C-i)\mu_p \right) z - \lambda_{hp} z^2 - \right.$$

$$\left. (C-i)\mu_p + (C-i)(1-z)\mu_Q \right] \Pi_i(z) =$$

$$(i+1)\mu_v z \Pi_{i+1}(z) + \mu_Q z(1-z) \Pi_i'(z) +$$

$$\left[z^2 \lambda_{np} + z(C-i)\mu_p - z(\lambda_{nv} + \lambda_{hv} + \lambda_{np}) - (C-i)\mu_p \right] \sum_{j=0}^{C-i-1} P_{i,j} z^j +$$

$$\mu_p(1-z) \sum_{j=0}^{C-i-1} j P_{i,j} z^j - \mu_Q(1-z) \sum_{j=0}^{C-i+1} j P_{i,j} z^j +$$

$$z(\lambda_{nv} + \lambda_{hv}) \sum_{j=0}^{C-i} P_{i-1,j} z^j + \mu_Q(1-z)(C-i) \sum_{j=0}^{C-i+1} P_{i,j} z^j \qquad (11)$$

Now, for $i = 0, 1, 2, \ldots, C$ we define

$$D_i(z) = \frac{\left[\lambda_{hp} + i\mu_v + (C-i)\mu_p \right] z - \lambda_{hp} z^2}{\mu_Q z(1-z)} +$$

$$\frac{-(C-i)\mu_p + \mu_Q(C-i)(1-z)}{\mu_Q z(1-z)} \qquad (12)$$

So, we get a system of ordinary differential equations

$$\Pi_i'(z) = \frac{(i+1)\mu_v}{z(1-z)\mu_Q} \Pi_{i+1}(z) - D_i(z)\Pi_i(z) + b_i(z) \qquad (13)$$

where

$$b_i(z) = \left\{ \left[z^2 \lambda_{np} + z(C-i)\mu_p - z(\lambda_{nv} + \lambda_{hv} + \lambda_{np}) - \right. \right.$$

$$\left. (C-i)\mu_p \right] \sum_{j=0}^{C-i-1} P_{i,j} z^j + \mu_p(1-z) \sum_{j=0}^{C-i-1} j P_{i,j} z^j -$$

$$\mu_Q(1-z) \sum_{j=0}^{C-i+1} j P_{i,j} z^j + z(\lambda_{nv} + \lambda_{hv}) \sum_{j=0}^{C-i} P_{i-1,j} z^j +$$

$$\left. \mu_Q(1-z)(C-i) \sum_{j=0}^{C-i+1} P_{i,j} z^j \right\} \bigg/ z(1-z)\mu_Q$$

It is obvious that the state probabilities with indices $i < 0$ and $j < 0$ are taken equal to zero, and also if the upper index of a summation is less than the lower index its contribution to $b_i(z)$ is zero.

The system of (13) is a nonhomogeneous differential system of first order of the typical form

$$\frac{dX}{dz} = AX + B \qquad B = [b_0, b_1, \ldots, b_C]$$

where the elements of the matrices A and B are continuous functions of z in the region $|z| < 1$.

Since A is a bidiagonal matrix we can construct

$C = \int_{z_0}^{z} A(z) dz$ another bidiagonal matrix satisfying $AC \equiv CA$ and then we can apply Picard's theorem [5] to obtain an approximate solution for the functions $\Pi_i(z)$ or we can use numerical methods leading to approximate solutions, too.

An initial condition of the system (13) always exists, because when $z = 1$ the system becomes

$$i\mu_v \Pi_i(1) - (i+1)\mu_v \Pi_{i+1}(1) = (\lambda_{nv} + \lambda_{hv}) \left[\sum_{j=0}^{C-i} P_{i-1,j} - \sum_{j=0}^{C-i-1} P_{i,j} \right] \quad (15)$$

and can be solved by Cramer's rule.

After taking an approximate solution of the generating functions the state probabilities theoretically can be evaluated by the property

$$\frac{1}{j!} \left. \frac{d^j \Pi_i(z)}{dz^j} \right|_{z=0} = P_{i,j} \quad (16)$$

and then the performance measures are given by
i) The blocking probabilities $P_B = P_{Bnv} = P_{Bhv} = P_{Bnp}$ of any voice call or new packets is

$$P_B = \sum_{i=1}^{C} P_{i,C-i} \quad (17)$$

and
ii) The mean number of packets E(j) in the system is

$$E(j) = \sum_{i=0}^{S} \Pi_i'(1) \quad (18)$$

and the mean time E(T) of packets in the system is

$$E(j) = (\lambda_{hp} + \lambda_{np}) E(T) \quad (19)$$

The analysis of the above system seems to be rather disappointing because of its complexity and perhaps intractable for large C. But for some special operational rules and system structures closed form solutions can easily attained, thus two-dimensional traffic models are generally acceptable for mobile systems analysis. One such case is described in the following section.

III. A SPECIAL TWO-DIMENSIONAL MODEL

We consider a system with two types of traffic (voice calls and data packets) having access to a service facility, which is formed by a set of C channels plus a buffer of size K-C, thus our system is one of finite capacity K. The arrival rates are λ_1 and λ_2 respectively and the channel holding time in a cell (the time a call or a packet occupies a channel while its terminal is crossing the cell) follows exponential distributions for both types of traffic with means $1/\mu_1$ and $1/\mu_2$ respectively. Any type of arrival has access to any facility but voice can preempt the service of data which return to the queue next to the last voice arrival. Thus we have a system with preemptive priority in the C channels and Head-of-the-Line (HOL) priority in the queue with voice priority over data. A call in such a system is blocked only if there are already K calls in the system while a data packet is blocked if the system is full in anyway. Moreover, any type of traffic

must leave the queue after a finite time because the vehicle has to leave the cell.

We assume that the time T_Q a voice call or a data packet is allowed to spend in the queue (the dwell time) follows an exponential distribution with mean $1/\mu_Q$ dependent mainly on the system structure i.e. the cell length and the vehicle speed. The operation rule of our system is described in Fig.2 and the state diagrams are given in Fig.3a, Fig.3b, where the variables i and j denote the number of calls and packets in the system.
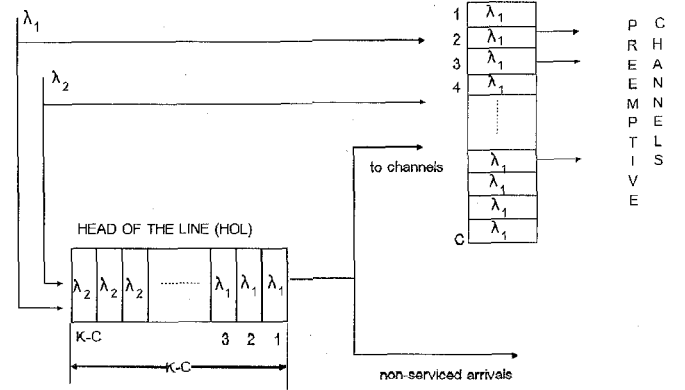

Fig. 2. The system operation rule for i>C and j=0,1,...,K-i (a special two-dimensional case).

For the above priority rule the state balanced equations are given by

$$[\lambda_1 + \lambda_2 + j\mu_2 + i\mu_1] P_{i,j} = \lambda_1 P_{i-1,j} + (i+1)\mu_1 P_{i+1,j} +$$
$$\lambda_2 P_{i,j-1} + (j+1)\mu_2 P_{i,j+1} \quad i+j < C \quad (20)$$

$$[\lambda_1 + \lambda_2 + i\mu_1 + (C-i)\mu_2 + (j-C+i)\mu_Q] P_{i,j} =$$
$$\lambda_1 P_{i-1,j} + \lambda_2 P_{i,j-1} + (i+1)\mu_1 P_{i+1,j} +$$
$$(C-i)\mu_2 P_{i,j+1} + (j-C+1+i)\mu_Q P_{i,j+1} \quad (21)$$
$$\text{for } i < C \quad K > i+j \geq C$$

$$[\lambda_1 + \lambda_2 + C\mu_1 + (i-C)\mu_Q + j\mu_Q] P_{i,j} = \lambda_1 P_{i-1,j} + \lambda_2 P_{i,j-1} +$$
$$C\mu_1 P_{i+1,j} + (i+1-C)\mu_Q P_{i+1,j} + (j+1)\mu_Q P_{i,j+1} \quad (22)$$
$$\text{for } i \geq C \quad j = 0,1,2,...,K-i-1$$

$$[\lambda_1 + C\mu_1 + (i-C)\mu_Q + j\mu_Q] P_{i,j} =$$
$$\lambda_1 P_{i-1,j} + \lambda_2 P_{i,j-1} + \lambda_1 P_{i-1,j+1} \text{ for } i > C \quad i+j = K \quad (23)$$

$$[\lambda_1 + i\mu_1 + (C-i)\mu_2 + (j-C+i)\mu_Q] P_{i,j} =$$
$$\lambda_1 P_{i-1,j} + \lambda_2 P_{i,j-1} + \lambda_1 P_{i-1,j+1} \text{ for } i < C \quad i+j = K \quad (24)$$
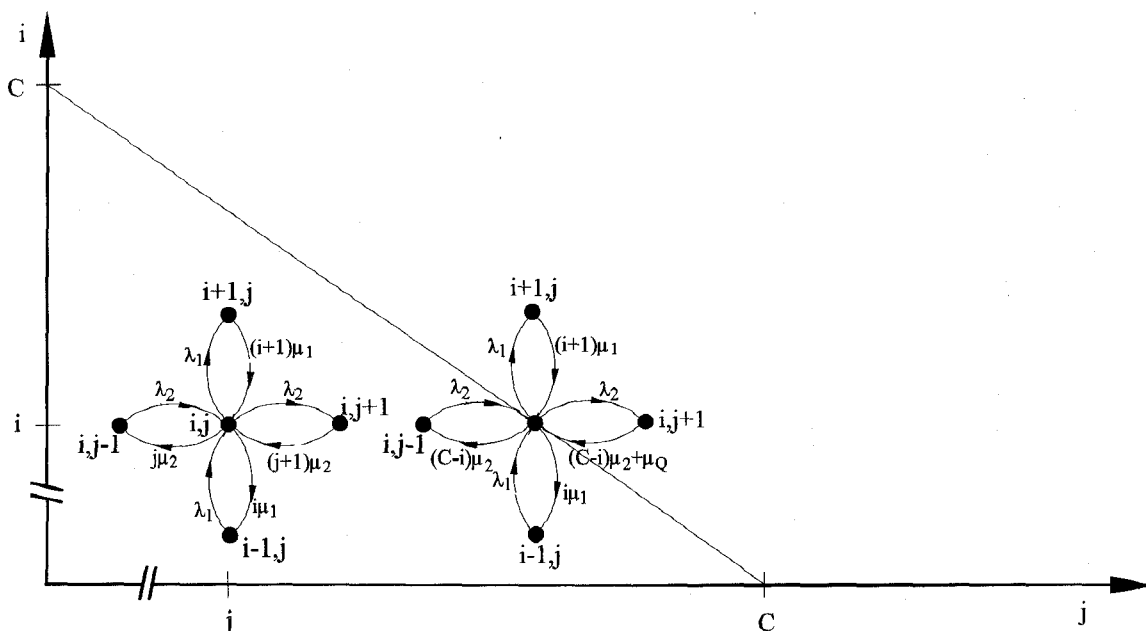
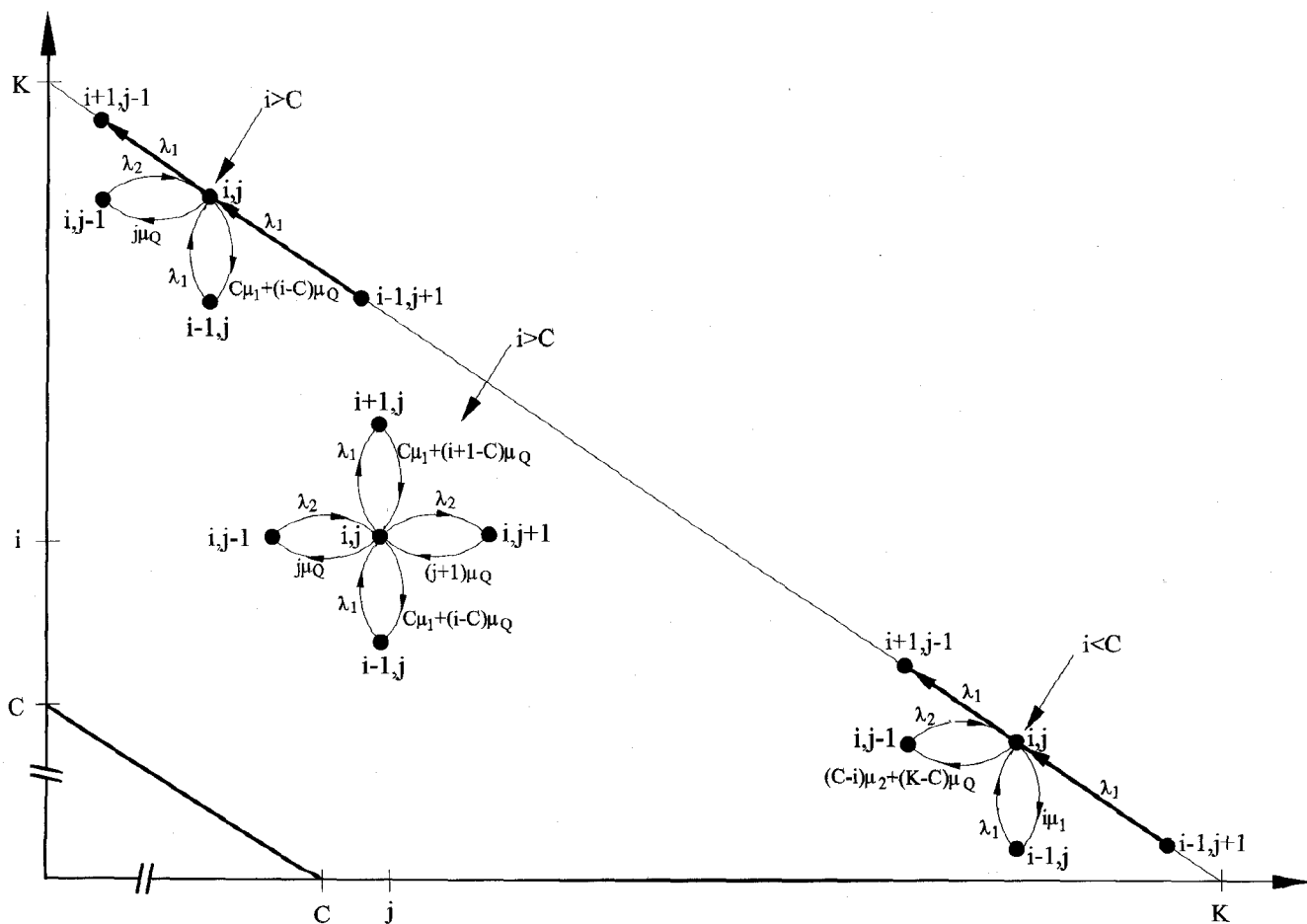Fig. 3a. Two-dimensional state diagram for a special two-dimensional case $i+j \leq C$, $j<C$.

Fig. 3b. Two-dimensional state diagram for a special two-dimensional case $C<i+j \leq K$.

In these equations there are $(K+1)(K+2)/2$ unknowns to be determined, the probabilities $P_{i,j}$,for $i=0,1,2...K$ and $j=0,1,2,...,K$. But there are only $K(K+1)/2$ independent equations derived from the system of (20)-(24) thus we need $K+1$ more. We observe that since voice possesses preemptive priority over data in the service facility and HOL priority in the queue, the probabilities $P_{i,0}$ for $i=0,1,2,...,K$ are given by the M/M/C/K formulae modified in purpose of covering the mobility in queue i.e.

$$P_{i,0} = P_{0,0}\frac{\rho^i}{i!} \qquad i \le C \tag{25}$$

$$P_{i,0} = P_{0,0}\frac{\rho^C}{C!}\left(\frac{\lambda_1^{i-C}}{\prod_{n=1}^{i-C}(C\mu_1 + n\mu_Q)}\right) \quad C < i \le K$$

where $\rho = \dfrac{\lambda_1}{\mu_1}$

Moreover, we can use the normalising condition

$$\sum_i^K \sum_j^K P_{i,j} = 1 \quad i = 0,1,...,K \quad j = 0,1,...,K \tag{26}$$

The system of (20)-(26) is now sufficient for the evaluation of the state probabilities $P_{i,j}$. After the evaluation of these probabilities the main performance parameters are defined to be :

i) the voice and data blocking probabilities (the probabilities for the system being occupied) which are given respectively by

$$P_{B1} = P_{K,0} \tag{27}$$

$$P_{B2} = \sum_{i=0}^K P_{i,K-i} \tag{28}$$

ii) the mean number of voice calls in the system which is given by

$$E(N_1) = \sum_{i=0}^K iP_{i,j} \tag{29}$$

and the mean number of data packets in the system which is

$$E(N_2) = \sum_{i=0}^K \sum_{j=0}^{K-i} jP_{i,j} \tag{30}$$

Since the mobility in queue is included into the state probabilities we can use Little's formula to evaluate the

average time in the system as well as the average time in the queue, (in order to receive service), that is

$$T_1\lambda_1 = E(N_1) = \lambda_1\left(\frac{1}{w_1} + \frac{1}{\mu_1}\right) \tag{31}$$

and

$$T_2\lambda_2 = E(N_2) = \lambda_2\left(\frac{1}{w_2} + \frac{1}{\mu_2}\right) \tag{32}$$

and finally

iii) the failure probabilities (the probabilities of entering the queue but finally leaving the cell without receiving service) which are equal to

$$P_{F1} = \sum_{i=C}^K P_{i,j}P_{Q1} \tag{33}$$

$$P_{F2} = \sum_{i=0}^K \sum_{j=C-i+1}^{K-i} P_{i,j}P_{Q2} \tag{34}$$

The probability of leaving the cell P_Q is the probability that the dwell time T_Q in the cell is less than the waiting time W in the queue. Assuming that the variable W follows also an exponential distribution with mean $1/w$ we have

$$P_{Q1} = Prob(T_Q < W_1) = \int_0^\infty [1 - F_{W1}(t)]f_{T_Q}(t)dt =$$

$$= \int_0^\infty e^{-w_1 t}f_{\mu_Q}(t)dt = \frac{\mu_Q}{w_1 + \mu_Q} \tag{35}$$

and $P_{Q2} = Prob(T_Q < W_2) = \int_0^\infty [1 - F_{W2}(t)]f_{T_Q}(t)dt =$

$$= \int_0^\infty e^{-w_2 t}f_{T_Q}(t)dt = \frac{\mu_Q}{w_2 + \mu_Q} \tag{36}$$

where $1/w_1$ and $1/w_2$ are given by (31), (32).

The evaluation of the above measures defines completely the system performance.

IV. CONCLUSION

A traffic analysis of a mixed media cellular system has been presented using two-dimensional state diagrams. The characteristic mobility in the queue state leads to complex differential equations of generating functions with non-constant matrix elements. But for specific structures and operation rules the performance measures can be obtained as functions of the arrival rates (λ) channel holding time rates (μ) and the system structure (μ_Q).

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