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VARIABLE REDUCTION METHOD IN ROUTING PROBLEMS

Indexing terms: Computers, Computer communications, Routing algorithms

A variable reduction method is used in the mathematical formulation of routing problems where independent variables obey linear constraints. In this respect the first and second derivatives of the reduced objective function are evaluated. The Hessian matrix results are diagonal and therefore Newton's method can easily be applied.

Introduction: In routing problems in computer networks the routing strategy which minimises the average time delay D is attained assuming that the topology and the link capacities are given. The average time delay D for the one-destination case is given by the expression[1]

$$D = \frac{1}{\gamma} \sum_{(i,k)} \frac{f_{ik}}{\mu C_{ik} - f_{ik}} = \frac{1}{\gamma} \sum_{(i,k)} \frac{t_i \varphi_{ik}}{\mu C_{ik} - t_i \varphi_{ik}} \tag{1}$$

where (i, k) denotes the directed link connecting the i and k nodes, f_{ik} is the flow in link (i, k) (mess/s), $0 \leq f_{ik} \leq C_{ik}$, t_i is the total flow at node i (mess/s), $0 \leq t_i$, C_{ik} is the constant capacity of link (i, k) (bit/s), $0 \leq C_{ik}$, $1/\mu$ is the average message length (bit/mess), $\gamma = \sum_i \gamma_i$ is the total arrival flow in the network (mess/s), $0 \leq \gamma_i$, and φ_{ik} is the fraction of t_i routed through link (i, k), $0 \leq \varphi_{ik} \leq 1$; moreover it is evident that $\varphi_{ik} = f_{ik}/t_i$.

The independent variables in eqn. 1 are the flow fractions φ_{ik} which obey the constraint

$$\sum_k \varphi_{ik} = 1 \tag{2}$$

i.e. the law of flow conservation at node i.

Methods of nonlinear programming, and especially Newton's method, have been used extensively for the solution of the above stated problem.[2,3] However, a difficulty in that method is the construction and inversion of the matrix of second derivatives, i.e. the Hessian matrix. Since the mathematical formula which gives the second-order derivatives of eqn. 1 is very complex, there is a tendency to consider the Hessian and therefore its inverse to be diagonal and consequently tractable in computations. It is to be observed, however, that eqn. 2 denotes a linear relationship among the independent variables at each node of the network and therefore the Hessian matrix cannot be diagonal; nor can it be invertible, because there is a linear dependence among its columns.[4] This means that Newton's method cannot be straightforwardly applied.

Now, the linearity of eqn. 2 leads us to try a variable reduction method to eliminate some of the independent variables. In this letter a mathematical model for the routing problem is formulated based on a variable reduction method. The obtained Hessian matrix is now diagonal and therefore Newton's method can easily be applied on the modified objective function.

Mathematical analysis: Eqn. 2, on one hand, denotes a linear relationship among the independent variables φ_{ik} at each node i of the network and, on the other hand, permits one of these variables to be expressed as a linear combination of the others. Assuming two outgoing links (i, m) and (i, k) at node i, eqn. 2 becomes

$$\varphi_{ik} + \varphi_{im} = 1 \Rightarrow \varphi_{ik} = 1 - \varphi_{im} \tag{3}$$

It is to be noted that eqn. 3 holds even if there are more than two variables. Now, calling x_{im} the independent variable φ_{im} and x_{ik} the dependent variable φ_{ik}, eqn. 3 becomes

$$x_{ik} = \varphi_{ik} = 1 - \varphi_{im} = 1 - x_{im} \tag{4}$$

and therefore eqn. 1 is

$$D = D_1 + D_2 = \frac{1}{\gamma} \sum_{(i,m)} \frac{t_i x_{im}}{\mu C_{im} - t_i x_{im}} + \frac{1}{\gamma} \sum_{(i,k)} \frac{t_i x_{ik}}{\mu C_{ik} - t_i x_{ik}}$$
$$= \frac{1}{\gamma} \sum_{(i,m)} \frac{t_i x_{im}}{\mu C_{im} - t_i x_{im}} + \frac{1}{\gamma} \sum_{(i,k)} \frac{t_i\left(1 - \sum^{k-1} x_{im}\right)}{\mu C_{ik} - t_i\left(1 - \sum^{k-1} x_{im}\right)} \tag{5}$$

where D is the sum of two terms D_1 and D_2, each one being a function only of the independent variable $x_{im} = \varphi_{im}$ and the dependent variable $x_{ik} = \varphi_{ik}$, respectively. Moreover, it is evident that

$$0 \leq x_{im} \leq 1 \tag{6}$$

To minimise D we have to calculate the first and second derivatives with respect to independent variables x_{im}. Considering only two variables at the moment and taking the first derivative of D with respect to x_{im} we have

$$\frac{\partial D}{\partial x_{im}} = \frac{\partial D_1}{\partial x_{im}} + \frac{\partial D_2}{\partial x_{im}} = \frac{\partial D_1}{\partial x_{im}} + \frac{\partial D_2}{\partial x_{ik}} \frac{\partial x_{ik}}{\partial x_{im}} \tag{7}$$

Eqn. 7 because of eqn. 4 becomes

$$\frac{\partial D}{\partial x_{im}} = \frac{\partial D_1}{\partial x_{im}} - \frac{\partial D_2}{\partial x_{ik}} \tag{8}$$

Now, turning back to the variables φ_{ik}, φ_{im} we have

$$\frac{\partial D}{\partial x_{im}} = \frac{\partial D_1}{\partial x_{im}} - \frac{\partial D_2}{\partial x_{ik}} = \frac{\partial D_1}{\partial \varphi_{im}} - \frac{\partial D_2}{\partial \varphi_{ik}} = \frac{\partial D}{\partial \varphi_{im}} - \frac{\partial D}{\partial \varphi_{ik}} \tag{9}$$

Gallager[1] has shown that

$$\frac{\partial D}{\partial \varphi_{im}} = t_i\left(D'_{im} + \frac{\partial D}{\partial \gamma_m}\right) = t_i \delta_{im} \tag{10}$$

where D'_{im} is the first derivative of D with respect to f_{im} and $\partial D/\partial \gamma_m$ is the first derivative of D with respect to the incoming flow at node m. Therefore eqn. 9 becomes

$$\frac{\partial D}{\partial x_{im}} = t_i(\delta_{im} - \delta_{ik}) \tag{11}$$

To generalise our results we consider now more than two outgoing links at node i. Eqn. 3 becomes $\varphi_{ik} + \varphi_{im} + \varphi_{in} + \ldots = 1$. That, however, does not affect our results because the condition given by eqn. 8 holds always. Indeed it is clear that the derivative of D with respect to each independent variable x_{im}, x_{in}, ..., is equal to the difference of the derivative of D with respect to the corresponding variables φ_{im}, φ_{in}, ..., minus the derivative of D with respect to the dependent variable φ_{ik} which has been expressed as a linear combination of φ_{im}, φ_{in}, ... In other words, we obtain a system of equations of the form of eqn. 9.

The second-order derivatives can be evaluated from eqn. 9:

$$\frac{\partial^2 D}{\partial(x_{im})^2} = \frac{\partial^2 D}{\partial(\varphi_{im})^2} - \frac{\partial^2 D}{\partial\varphi_{ik}\,\partial\varphi_{im}} \tag{12}$$

From eqns. 8 and 19 of a previous letter,[4]

$$\frac{\partial^2 D}{\partial\varphi_{ik}\,\partial\varphi_{im}} = -\frac{\partial^2 D}{\partial(\varphi_{im})^2} \tag{13}$$

and therefore eqn. 12 becomes

$$\frac{\partial^2 D}{\partial(x_{im})^2} = 2\,\frac{\partial^2 D}{\partial(\varphi_{im})^2} \tag{14}$$

This result gives a reduced Hessian matrix with diagonal elements twice the diagonal elements of the corresponding ones on the initial Hessian matrix. These elements are expressed[3] by the relationship

$$\frac{\partial^2 D}{\partial(\varphi_{im})^2} = t_i^2(D''_{im} + R_m) \tag{15}$$

where D''_{im} is the second derivative of D with respect to f_{im} and R_m is

$$R_m = \sum_l (\varphi_{ml})^2 D''_{ml} + \left(\sum_{l'} \varphi_{ml'}\,\delta_{ml'}\right)\left(\sum_l \frac{\varphi_{ml}}{\delta_{ml}} R_l\right)$$

with l and l' downstream neighbour nodes of node m.

It is to be noted that the reduced Hessian matrix is pure diagonal, since $x_{im}, x_{in}, \ldots$, are independent of each other. Hence all the derivatives of the form $(\partial^2 D)/(\partial x_{im}\,\partial x_{in}) = (\partial^2 D)/(\partial\varphi_{im}\,\partial\varphi_{in})$ are equal to zero.

Now, Newton's method can easily be applied to the modified routing problem of eqns. 5 and 6. According to that method the function D can be approximated by a series

$$D(\varphi) = D(\varphi_0) + [\nabla D(\varphi_0)](\varphi - \varphi_0) + \tfrac{1}{2}(\varphi - \varphi_0)^T[\nabla^2 D(\varphi_0)](\varphi - \varphi_0) \tag{16}$$

where $\nabla D(\varphi)$ is the gradient of D and $\nabla^2 D(\varphi)$ the Hessian matrix. The minimum occurs at[5]

$$\varphi^{k+1} = \varphi^k - a[\nabla^2 D(\varphi^k)]^{-1}[\nabla D(\varphi^k)]^T \tag{17}$$

where $[\;]^{-1}$ denotes matrix inverse, $a > 0$ is the step size of the algorithm and k denotes the iteration number.

Using eqns. 11, 14 and 15, eqn. 17 now becomes

$$\varphi_{im}^{k+1} = \varphi_{im}^k - a\,\frac{\delta_{im} - \delta_{ik}}{2t_i(D''_{im} + R_m)} \qquad 0 \le \varphi_{im} \le 1 \tag{18}$$

Computer applications of the above described procedure show that the method gives very good performance. Moreover, a comparison with the method used by Chen[3] has shown that our algorithm gives better results.

Conclusion: It is shown that the variable reduction method described here can be used for the mathematical formulation of routing problems where the independent variables obey linear constraints. The reduced Hessian matrix is diagonal and therefore Newton's method can easily be applied.

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USE OF BILINEAR INTEGRATOR IN DESIGN OF FULLY DIFFERENTIAL BIQUADS

Indexing terms: Circuit theory and design, Switched-capacitor networks

The design of a bilinear integrator in fully differential form introduces new possibilities in the design of biquad circuits. The new circuits presented here have no simple nondifferential counterparts, are stray-insensitive, have small sensitivities and require capacitor values in acceptable limits.

Introduction: Fully differential (FD) configurations have been widely used[1–7] because they have several advantages when compared to simple nondifferential configurations. The main advantages are improvements in power supply noise rejection and in the effects of charge injection via the switches.[1,2]

Beyond these advantages, FD structures present additional design flexibility owing to the availability of outputs of both signs. Also, a bilinear switched-capacitor (SC) integrator in FD form can be realised.[5,6] In this letter FD biquad structures are presented using the bilinear integrator.

FD SC integrators: In Fig. 1 three FD (SC) integrators are shown.[3–6] The first, Fig. 1*a*, is a bilinear FD integrator. The

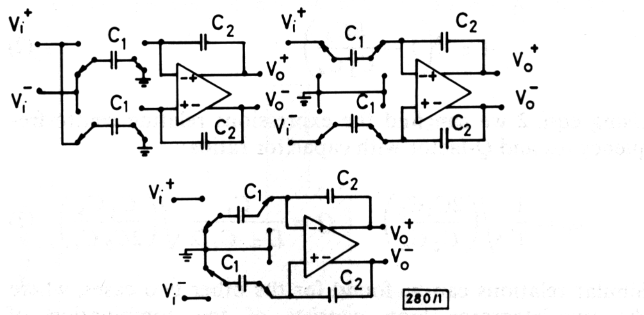

Fig. 1 *Fully differential integrators*
a Bilinear
b Inverting
c Noninverting

second, Fig. 1*b*, and the third, Fig. 1*c*, are the FD counterparts of the well known nondifferential inverting and noninverting integrators.[8,9] These three integrators realise the following transfer functions:

$$H_a = \mp\frac{C_1}{C_2}\frac{1 + z^{-1}}{1 - z^{-1}} \qquad H_b = \mp\frac{C_1}{C_2}\frac{1}{1 - z^{-1}}$$

$$H_c = \pm\frac{C_1}{C_2}\frac{z^{-1}}{1 - z^{-1}} \tag{1}$$

The '±' sign in eqns. 1 has the meaning that, by exchanging the polarity of the input signal, the function sign is inverted.

FD SC biquads: The design of an SC biquad is based on the realisation of the two integrator loop.[8–11]

The bilinear integrator can be combined with the inverting or noninverting integrator to form the two integrator loop needed for the biquad realisation.

Also, two bilinear integrators can be combined for the same